

# QuiTRU: Design Secure Variant of Ntruencrypt Via a New Multi-Dimensional Algebra

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**Abstract:** The NTRU public-key cryptosystem is based on time complexity and efficient computations. Many researchers were stimulated to improve NTRU performance by changing mathematical structure with new algebraic structures and replacing the truncated polynomial ring, such as OTRU, QTRU. In this paper, we proposed QuiTRU as a new version of the NTRU. It's a five-dimensional cryptosystem based on a new algebraic structure. As well, QuiTRU was compared with NTRU, QTRU, and OTRU.

**Keywords:** NTRU, QTRU, OTRU, QuiTRU.

## 1 Introduction

The first NTRU version was suggested by [1]. It has been assessed recently as the fastest public-key cryptosystems, its operations are in the truncated polynomial ring with coefficients in  $Z$ . Many studies were published to improve NTRU, some of them were focused on security improvement through the replacement of the original ring. [2] presented a generalization of NTRU by the ring of polynomials over the binary field  $F_2$  which is called CTRU. [3] presented an analog of NTRU called MaTRU; this system operates in the ring of  $k \times k$  matrices of polynomials in  $Z[x]/(x^n - 1)$ . [4] presented a system called QTRU that relies on quaternion algebra. [5] presented a cryptosystem called OTRU, which depends on the octonion algebra. [6, 7] introduced ETRU depends on Eisenstein integer ring  $Z[\omega]$ . [8] proposed a new multidimensional public key cryptosystem called CQTRU that operates in commutative quaternion algebra. [9, 10, 11], presented new cryptosystems, called HXDTRU and BITRU, based on their hexadecenion and binary algebras. [12, 13] introduced a new NTRU-like cryptosystem that depends on the bi-cartesian algebra; they called it BCTRU. [14] introduced a new NETRU cryptosystem that operates over the ring  $M = M_k(Z_p)[T, x]/(X^n - I_{k \times k})$  of  $k \times k$  matrices of elements in the ring

$R = Z_p[T, x]/(x^n - 1)$ . [15] introduced a new NTRU-analog cryptosystem using multidimensional carternion algebra called  $QOB_{TRU}$ . [16] proposed a new multi-dimensional public key cryptosystem, called NTRTE which relies on a commutative quaternion algebra with a new structure. [17] presented QMTRU with a new mathematical structure as an improvement for QTRU. This paper designed a new version of NTRU, called QuiTRU depends on the new algebra, namely HH-Real algebra with a new mathematical structure. This paper is structured as follows. HH-Real algebra was introduced in Section 2. The QuiTRU cryptosystem is defined in section 3, the security analysis of QuiTRU is discussed in section 4. The comparison is carried out in Section 5 with QuiTRU, NTRU, QTRU, and OTRU. Finally, some conclusions are in section 6.

## 2 HH-REAL ALGEBRA

The following is the describes of a new HH-Real algebra, which is a five-dimensional vector space over the real number  $\mathbb{R}$ :

$$HH = \{a_0\tau_0 + a_1\tau_1 + a_2\tau_2 + a_3\tau_3 + a_4\tau_4 \mid a_0, \dots, a_4 \in \mathbb{R}\}.$$

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It is called real HH-Real algebra with basis  $\{\tau_0, \tau_1, \tau_2, \tau_3, \tau_4\}$  such that  $\tau_0 = (1, 0, 0, 0, 0)$ ,  $\tau_1 = (0, 1, 0, 0, 0)$ ,  $\tau_2 = (0, 0, 1, 0, 0)$ ,  $\tau_3 = (0, 0, 0, 1, 0)$ ,  $\tau_4 = (0, 0, 0, 0, 1)$ .

Let  $p_0 = a_0\tau_0 + a_1\tau_1 + a_2\tau_2 + a_3\tau_3 + a_4\tau_4$  and  $p_1 = b_0\tau_0 + b_1\tau_1 + b_2\tau_2 + b_3\tau_3 + b_4\tau_4 \in \text{HH}$ , then the operations on this algebra are defined as follows: The addition:

$$p_0 + p_1 = (a_0 + b_0)\tau_0 + (a_1 + b_1)\tau_1 + (a_2 + b_2)\tau_2 + (a_3 + b_3)\tau_3 + (a_4 + b_4)\tau_4.$$

The multiplication:

$$p_0 * p_1 = (a_0b_0)\tau_0 + (a_1b_1)\tau_1 + (a_2b_2)\tau_2 + (a_3b_3)\tau_3 + (a_4b_4)\tau_4$$

where  $*$  is the HH-Real algebra product. This multiplication is associative and commutative. The multiplicative inverse is defined as follows:

$$p_0^{-1} = c_0\tau_0 + c_1\tau_1 + c_2\tau_2 + c_3\tau_3 + c_4\tau_4$$

Such that:

$$c_0 = \frac{1}{a_0}, c_1 = \frac{1}{a_1}, c_2 = \frac{1}{a_2}, c_3 = \frac{1}{a_3}, c_4 = \frac{1}{a_4} \text{ and } a_0, \dots, a_4 \neq 0.$$

Consider the rings  $\mathcal{K} = \mathbb{Z}[x]/(x^N - 1)$ ,  $\mathcal{K}_p = \mathbb{Z}_p[x]/(x^N - 1)$ , and  $\mathcal{K}_q = \mathbb{Z}_q[x]/(x^N - 1)$ . The HH-Real algebras  $\mathcal{D}$ ,  $\mathcal{D}_p$ , and  $\mathcal{D}_q$  are defined as follows:

$$\begin{aligned} \mathcal{D} &= \{f_0\tau_0 + f_1\tau_1 + f_2\tau_2 + f_3\tau_3 + f_4\tau_4 \mid f_0, \dots, f_4 \in \mathcal{K}\} \\ \mathcal{D}_p &= \{f_0\tau_0 + f_1\tau_1 + f_2\tau_2 + f_3\tau_3 + f_4\tau_4 \mid f_0, \dots, f_4 \in \mathcal{K}_p\} \\ \mathcal{D}_q &= \{f_0\tau_0 + f_1\tau_1 + f_2\tau_2 + f_3\tau_3 + f_4\tau_4 \mid f_0, \dots, f_4 \in \mathcal{K}_q\}. \end{aligned}$$

### 3 The QuiTRU Cryptosystem

#### 3.1 Parameter Creation

As in NTRU, QuiTRU has three parameters  $N, p, q$  defined in the same manner, and the subsets  $(\mathcal{L}_F, \mathcal{L}_G, \mathcal{L}_J, \mathcal{L}_R, \mathcal{L}_\phi, \mathcal{L}_M) \subset \mathcal{D}$  are defined in Table 1.

**Table 1:** Subsets of QuiTRU

Notation	Definition
$\mathcal{L}_F$	$\{f_0(x)\tau_0 + f_1(x)\tau_1 + f_2(x)\tau_2 + f_3(x)\tau_3 + f_4(x)\tau_4 \in \mathcal{D} \mid f_0(x), \dots, f_4(x) \in \mathcal{K} \text{ satisfy } \ell(d_f, d_f - 1)\}$
$\mathcal{L}_G$	$\{g_0(x)\tau_0 + g_1(x)\tau_1 + g_2(x)\tau_2 + g_3(x)\tau_3 + g_4(x)\tau_4 \in \mathcal{D} \mid g_0(x), \dots, g_4(x) \in \mathcal{K} \text{ satisfy } \ell(d_g, d_g)\}$
$\mathcal{L}_J$	$\{j_0(x)\tau_0 + j_1(x)\tau_1 + j_2(x)\tau_2 + j_3(x)\tau_3 + j_4(x)\tau_4 \in \mathcal{D} \mid j_0(x), \dots, j_4(x) \in \mathcal{K} \text{ satisfy } \ell(d_j, d_j)\}$
$\mathcal{L}_R$	$\{r_0(x)\tau_0 + r_1(x)\tau_1 + r_2(x)\tau_2 + r_3(x)\tau_3 + r_4(x)\tau_4 \in \mathcal{D} \mid r_0(x), \dots, r_4(x) \in \mathcal{K} \text{ satisfy } \ell(d_r, d_r)\}$
$\mathcal{L}_\phi$	$\{\phi_0(x)\tau_0 + \phi_1(x)\tau_1 + \phi_2(x)\tau_2 + \phi_3(x)\tau_3 + \phi_4(x)\tau_4 \in \mathcal{D} \mid \phi_0(x), \dots, \phi_4(x) \in \mathcal{K} \text{ satisfy } \ell(d_\phi, d_\phi)\}$
$\mathcal{L}_M$	$\{m_0(x)\tau_0 + m_1(x)\tau_1 + m_2(x)\tau_2 + m_3(x)\tau_3 + m_4(x)\tau_4 \in \mathcal{D} \mid \text{coefficients of } m_0(x), \dots, m_4(x) \in \mathcal{K} \text{ are the chosen modulo between } -p/2 \text{ and } p/2\}$

where  $\ell(d_x, d_y) = \{f \in \mathcal{K} \mid f \text{ has } d_x \text{ coeff. equal to } 1, d_y \text{ coeff. equal to } -1, \text{ and the rest are } 0\}$ .

#### 3.2 Key Generation

To generate the keys, first, we randomly choose  $F \in \mathcal{L}_F$ ,  $G \in \mathcal{L}_G$ , and  $J \in \mathcal{L}_J$ , such that F should be invertible under mod q and p. Let the inverses be denoted by  $F_q^{-1}$  and  $F_p^{-1}$ . An algorithm below illustrates public key generation in the proposed QuiTRU.

**Algorithm 1** The proposed QuiTRU: Keys Generation Process

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- Input:  $N, p, q, F, G, J$   
 Output: public key  $H$
- 1: Choose randomly  $F \in \mathcal{L}_F, G \in \mathcal{L}_G$ , and  $J \in \mathcal{L}_J$
  - 2: Compute  $F_q^{-1} = \text{inverse of } F \pmod{q}$
  - 3: Compute  $H = F_q^{-1} * G * J \pmod{q}$
  - 4: Return  $H$  as a public key and  $\{F, G, J\}$  is a set of the private keys
  - 5: end
- 

#### 3.3 Encryption

Initially, we choose  $\phi \in \mathcal{L}_\phi, R \in \mathcal{L}_R$ , to encrypt the message  $M \in \mathcal{L}_M$ , it should be convert to HH-Real algebra form. Then, the encrypted message is

$$E = p(H * \phi + R) + M \pmod{q}.$$

An algorithm below illustrates encryption in the proposed QuiTRU.

**Algorithm 2** The proposed QuiTRU: Encryption process

Input:  $N, p, q, F, G, J$   
 Output: The encryption  $E$  of message  $M$

- 1: chooses message  $M \in \mathcal{L}_M$
- 2: converts a message  $M$  into a HH-Real algebra form.
- 3: computes the ciphertext  $E = p(H * \Phi + R) + M \pmod{q}$
- 4: Return  $E$  as a ciphertext  $E = p(H * \Phi + R) + M \pmod{q}$ .
- 5: end

**3.4 Decryption**

Upon receiving the first user to the ciphertext, he/ she wants to decrypt it and recover the original message  $M$ . So, the decryption process can be explained by Algorithm (3).

**Algorithm 3** The proposed QuiTRU: Encryption process

Input:  $N, q, p, F, F_p^{-1}, E$   
 Output: A message  $M$

- 1: chooses message  $M \in \mathcal{L}_M$
- 2: Compute  $D = F * E \pmod{q}$
- 3: **for**  $i = 1$  to 5 **do**
- 4:     **for**  $i = 1$  to 5 **do**
- 5:         **if**  $D_2(i, j) \leq \frac{-p}{2}$  **then**
- 6:             Compute  $D_2(i, j) = D_2(i, j) + p$
- 7:         **else if**  $D_2(i, j) > p/2$  **then**
- 8:             Compute  $D_2(i, j) = D_2(i, j) - p$
- 9:         **end if**
- 10:     **end for**
- 11: **end for**
- 12: Return  $M = D_2$
- 13: end

**4 Security analysis**

There are many attacks that have been used to analyze NTRU and their variance cryptosystems. The most used one is the brute force attack, the attacker who know the public parameters and the public key  $H = F_q^{-1} * G * J \pmod{q}$  is tried to find the private key  $F$  from the set  $\mathcal{L}_F$  (or find the private keys  $G, J$  from the sets  $\mathcal{L}_G, \mathcal{L}_J$ ). This helps to find a decryption short key, or the random polynomials  $\phi, R$  from the set  $\mathcal{L}_\phi, \mathcal{L}_R$ , which leads to find the message. The size of the subsets  $\mathcal{L}_F, \mathcal{L}_G, \mathcal{L}_J, \mathcal{L}_\phi, \mathcal{L}_R$  is equal to

$$|\mathcal{L}_F| = \left( \frac{N!}{(d_f!)^2 (N - 2d_f)!} \right)^5, |\mathcal{L}_G| = \left( \frac{N!}{(d_g!)^2 (N - 2d_g)!} \right)^5,$$

$$|\mathcal{L}_J| = \left( \frac{N!}{(d_j!)^2 (N - 2d_j)!} \right)^5, |\mathcal{L}_\phi| = \left( \frac{N!}{(d_\phi!)^2 (N - 2d_\phi)!} \right)^5,$$

$$|\mathcal{L}_R| = \left( \frac{N!}{(d_r!)^2 (N - 2d_r)!} \right)^5.$$

Therefore, the size of the search space of finding the private keys  $G, J$  is

$$\frac{(N!)^{10}}{(d_g! d_j!)^{10} ((N - 2d_g)! (N - 2d_j)!)^5}.$$

Similarly, the size of the search space to find the polynomials  $R, \phi$  is

$$\frac{(N!)^{10}}{(d_r! d_\phi!)^{10} ((N - 2d_r)! (N - 2d_\phi)!)^5}.$$

Table 2 shows the security level of the private key space and message space according to the general parameters in QuiTRU and  $p = 3$

**Table 2:** The Space of the Private Key and the Message

$d_r$	$d_\phi$	$d_j$	$d_g$	$d_f$	$N$	Key space	Message space
5	5	12	12	12	107	$2.0843 \times 10^{301}$	$2.9627 \times 10^{159}$
10	10	20	20	20	107	$3.5056 \times 10^{407}$	$2.8397 \times 10^{266}$
10	10	12	12	12	149	$2.8413 \times 10^{339}$	$4.4184 \times 10^{297}$
20	20	25	25	25	149	$1.9727 \times 10^{542}$	$3.0923 \times 10^{476}$
18	18	18	18	18	167	$2.3168 \times 10^{466}$	$2.3168 \times 10^{466}$
22	22	27	27	27	167	$3.5184 \times 10^{597}$	$8.1583 \times 10^{529}$
18	18	20	20	20	211	$6.7356 \times 10^{544}$	$7.1192 \times 10^{506}$
22	22	34	34	34	211	$1.6819 \times 10^{758}$	$5.0344 \times 10^{580}$
18	18	20	20	20	257	$2.1743 \times 10^{582}$	$1.8873 \times 10^{540}$
24	24	24	24	24	257	$3.5936 \times 10^{660}$	$3.5936 \times 10^{660}$

**5 Comparing NTRU, QTRU, OTRU, and QuiTRU**

We now compare the proposed QuiTRU with the original NTRU, and QTRU, OTRU because the dimensional of QuiTRU between the dimensional of QTRU and OTRU according to some main criteria such as, level of security of the key and the message, efficiency.

**5.1 Level of Security**

In Tables 3 and 4, a comparison between the QuiTRU and NTRU, QTRU, and OTRU in terms of security level for the key and the message is introduced depending on the generic parameters. Thus, the comparison between key security and message security in the QuiTRU, NTRU, QTRU, and OTRU systems shows that the security level of QuiTRU is better than that of the NTRU, QTRU, and OTRU.

**Table 3:** Key Space in QuiTRU, NTRU, QTRU, and OTRU

Key Space			
QuiTRU	NTRU	QTRU	OTRU
$2.0843 \times 10^{301}$	$1.3549 \times 10^{30}$	$3.3696 \times 10^{120}$	$1.1355 \times 10^{241}$
$3.5056 \times 10^{407}$	$5.6817 \times 10^{40}$	$1.0421 \times 10^{163}$	$1.0859 \times 10^{326}$
$2.8413 \times 10^{339}$	$8.8176 \times 10^{33}$	$6.0452 \times 10^{135}$	$3.6544 \times 10^{271}$
$1.9727 \times 10^{542}$	$1.6963 \times 10^{54}$	$8.2799 \times 10^{216}$	$6.8557 \times 10^{433}$
$2.3168 \times 10^{466}$	$4.3300 \times 10^{46}$	$3.5153 \times 10^{186}$	$1.2357 \times 10^{373}$
$3.5184 \times 10^{597}$	$5.6837 \times 10^{59}$	$1.0436 \times 10^{239}$	$1.0891 \times 10^{478}$
$6.7356 \times 10^{544}$	$3.0397 \times 10^{54}$	$8.5378 \times 10^{217}$	$7.2895 \times 10^{435}$
$1.6819 \times 10^{758}$	$6.6463 \times 10^{75}$	$1.9513 \times 10^{303}$	$3.8076 \times 10^{606}$
$2.1743 \times 10^{582}$	$1.7129 \times 10^{58}$	$8.6085 \times 10^{232}$	$7.4107 \times 10^{465}$
$3.5936 \times 10^{660}$	$1.1358 \times 10^{66}$	$1.6681 \times 10^{264}$	$2.7825 \times 10^{528}$

**Table 4:** Message Space in QuiTRU, NTRU, QTRU, and OTRU

Message Space			
QuiTRU	NTRU	QTRU	OTRU
$2.9627 \times 10^{159}$	$8.8546 \times 10^{15}$	$6.1472 \times 10^{63}$	$3.7788 \times 10^{127}$
$2.8397 \times 10^{266}$	$4.4190 \times 10^{26}$	$3.8134 \times 10^{106}$	$1.4542 \times 10^{213}$
$4.4184 \times 10^{297}$	$5.8147 \times 10^{29}$	$1.1432 \times 10^{119}$	$1.3068 \times 10^{238}$
$3.0923 \times 10^{476}$	$4.4568 \times 10^{47}$	$3.9455 \times 10^{190}$	$1.5567 \times 10^{381}$
$2.3168 \times 10^{466}$	$4.3300 \times 10^{46}$	$3.5153 \times 10^{186}$	$1.2357 \times 10^{373}$
$8.1583 \times 10^{529}$	$9.7985 \times 10^{52}$	$9.2180 \times 10^{211}$	$8.4972 \times 10^{423}$
$7.1192 \times 10^{506}$	$4.8444 \times 10^{50}$	$5.5077 \times 10^{202}$	$3.0335 \times 10^{405}$
$5.0344 \times 10^{580}$	$1.1754 \times 10^{58}$	$1.9088 \times 10^{232}$	$3.6438 \times 10^{464}$
$1.8873 \times 10^{540}$	$1.0656 \times 10^{54}$	$1.2893 \times 10^{216}$	$1.6622 \times 10^{432}$
$3.5936 \times 10^{660}$	$1.1358 \times 10^{66}$	$1.6681 \times 10^{264}$	$2.7825 \times 10^{528}$

## 5.2 Mathematical Operations

In this subsection, a comparison of the QuiTRU, NTRU, QTRU, and the OTRU is described as shown in Table 5 according to the mathematical operations (polynomial addition and convolution (conv.) multiplication). Therefore, for key generation, encryption, and decryption of QuiTRU is performed faster than QTRU and OTRU, and slower than NTRU.

**Table 5:** Convolution multiplication and addition of QuiTRU, NTRU, QTRU, and OTRU

	QuiTRU	NTRU	QTRU	OTRU
<i>Key Creation</i>	10 conv. multiplications	one conv. multiplications	16 conv. multiplications	64 conv. multiplications
<i>Encryption</i>	five conv. multiplications, 10 polynomial addition	one conv. multiplications, one polynomial addition	16 conv. multiplications, four polynomial addition	64 conv. multiplications, eight polynomial addition
<i>Decryption</i>	20 conv. multiplications, 10 polynomial addition	two conv. multiplications, one polynomial addition	32 conv. multiplications, four polynomial addition	1024 conv. multiplications, eight polynomial addition

Table 6 compares the speed of QuiTRU, NTRU, QTRU, and OTRU based on Table 5, such that  $t$  is the time of convolution multiplication and  $t_1$  is the time of polynomial addition.

**Table 6:** Speed of QuiTRU, NTRU, QTRU, and OTRU

	QuiTRU	NTRU	QTRU	OTRU
<i>Speed</i>	$35t + 20t_1$	$4t + 2t_1$	$64t + 8t_1$	$1152t + 16t_1$

So, QuiTRU has higher security than NTRU, QTRU, and OTRU, but NTRU is faster.

## 6 Conclusion

We presented here a multi-dimensional analog of NTRU, called QuiTRU. It is based on newly generated HH-Real algebra. This system enjoys a very high security level compared to the three systems NTRU, QTRU, and OTRU. Also, QuiTRU can encrypt five messages in parallel. These messages may be created from a single source or several sources, because of HH-Real algebra has five dimensional. This feature may be important in some applications, for example, in electronic voting. The speed of QuiTRU is faster than QTRU and OTRU, and slower than NTRU.

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